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NEW INFORMATION ON B DECAYS TO CHARMLESS VP FINAL STATES [1]

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ABSTRACT

The decays of B mesons to charmless final states consisting of a vector meson (V) and a pseudoscalar meson (P) are analyzed within flavor SU(3). Predictions are compared with new data from the CLEO Collaboration. Dominant contributions to amplitudes and subdominant interfering terms are identified. Evidence is found for a specific penguin amplitude (contributing, for example, to $B^+ \to \rho^+ K^0$) at a level much higher than that implied by most explicit models. The validity of the conclusion $\cos\gamma < 0$, obtained through other analyses of $B \to VP$ decays, is examined here from a less model-dependent standpoint. It is found that several processes are consistent with $\cos\gamma < 0$ (or $\cos\alpha > 0$), and measurements are suggested which could make this conclusion more robust.

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I. INTRODUCTION

The decays of B mesons to charmless final states consisting of a vector meson (V) and a pseudoscalar meson (P) are of potentially great interest in the study of the weak interactions and CP violation. The decays $B^0 \to \rho^\mp \pi^\pm$ occur with a substantially greater combined branching ratio than $B^0 \to \pi^+\pi^-$, and hence may be useful in CP studies [1]. The decays $B^+ \to \omega\pi^+$, $B \to \omega K$, and $B \to \rho K$ can shed light on decay mechanisms by validating or falsifying specific models (most of which predict very low rates for $B \to (\omega,\rho)K$). Decays involving η and η' mesons are interesting because there exist models for the large observed rate for $B \to K\eta'$ which make specific predictions for $B \to K^*\eta$ and $B \to K^*\eta'$.

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In Ref. [2] we made a first attempt to classify $B \to VP$ decays in a model-independent manner, using only flavor SU(3) symmetry as expressed by a set of reduced amplitudes depicted in graphical form. The present article is an update of that work in the light of new experimental results from the CLEO Collaboration working at the Cornell Electron Storage Ring (CESR). These results include new branching ratios for charmless hadronic B decays to VP, where $V = K^*$, ρ, ω, and ϕ, and $P = (\pi,\ K)$ [3] or $B \to K^*(\eta, \eta')$ [4]. We shall also make use of some new results on $B \to PP$ decays [5], and will use the fact that no charge asymmetries have been seen in several processes [6].

One of the issues to which the new data are relevant is the phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ub}: $\gamma = \mathrm{Arg}(V_{ub}^*)$ in a standard convention. Data on $B \to K\pi$ decays have been used for several years to constrain this phase, both in relatively model-independent analyses [7, 8, 9, 10, 11, 12, 13, 14] and with the help of the factorization ansatz and models for form factors [5, 15, 16, 17, 18]. These studies tended to favor $\cos\gamma < 0$, consistent with determinations of CKM parameters which take realistic account of theoretical errors [19] but in some conflict with more optimistic estimates of these errors [20]. New factorization-based studies [15, 21] also find evidence for $\cos\gamma < 0$ in a number of $B \to VP$ processes. We wish to determine whether this conclusion holds in a less model-dependent context.

We find that *if final-state phase shifts are small*, consistent with the upper limits on charge asymmetries in several B decays to charmless final states [6]), then $B \to VP$ decays indeed favor $\cos\gamma < 0$ (or $\cos\alpha > 0$). Using the model-independent amplitudes obtained from the $B \to PV$ processes studied by CLEO, we can identify decay modes whose discovery or improved measurement would permit conclusions about $\cos\gamma$ and other CKM phases to be placed on a firmer footing. To exhibit the minimal experimental requirements for demonstrating interference-based constraints on $\cos\gamma$ or $\cos\alpha$, we perform this analysis independently of other constraints on CKM phases, mentioning them at the end of this paper.

A further result we obtain, in contrast to most explicit models, is that the penguin contributions to the decays $B \to (\omega, \rho)K$ are appreciable. In particular, the decay $B^+ \to \rho^+ K^0$ should be observable with a branching ratio in excess of 5×10^{-6}.

In Section II we recall some notation from Ref. [2] and tabulate experimental branching ratios and limits. Experimental results are taken from Ref. [3] unless not given there, in which case earlier limits [22] are quoted.

The crude nature of present experimental data on $B \to VP$ decays requires that a phenomenological analysis such as ours proceed in a somewhat roundabout way. Instead of simply presenting the results of an overall fit, which would fail to highlight the places where improved data are necessary, we focus first (in Sec. III) on the main amplitudes for which experimental data exist.

In Sec. IV we then see if deviations from rate relations predicted in the presence of a single amplitude can detect interference with one or more subdominant amplitudes (including electroweak penguin contributions). Sec. V summarizes the implications of the deduced amplitudes for other VP processes, tabulating the 90% confidence level (c.l.) ranges of predictions, both with and without tree-penguin interferences which might shed light on the sign of $\cos\gamma$ or $\cos\alpha$.

Sec. VI compares the results of our analysis with information about the size of amplitudes and the sign of tree-penguin interference in $B \to PP$ processes, and notes the prospects for obtaining complementary information from B_s–$\bar{B}_s$ mixing and $K^+ \to \pi^+ \nu\bar{\nu}$ decays. Sec. VII summarizes our arguments for negative $\cos\gamma$ and $\cos\alpha$, while an Appendix contains a short dictionary relating our invariant amplitudes in flavor SU(3) to quantities discussed in the factorization approximation, and discusses our findings in the context of the factorized-amplitude language.

II. NOTATION AND DESCRIPTION OF PROCESSES

In Table I we list some VP modes of nonstrange B mesons, their decomposition in terms of reduced amplitudes, and values or 90% c.l. upper limits for their branching ratios. We take amplitudes corresponding to the quark diagrams [23, 24] T (tree), C (color-suppressed), P (QCD-penguin), S (additional penguin involving flavor-SU(3)-singlet mesons), E (exchange), A (annihilation) and PA (penguin annihilation). The last three amplitudes, in which the spectator quark enters into the decay Hamiltonian, will be neglected here. Such contributions may be important in the presence of rescattering, for which tests exist [25]. Electroweak penguin contributions [26] are taken into account using the substitution [27]

$$T \to t \equiv T + P_{EW}^C \quad , \quad C \to c \equiv C + P_{EW} \quad ,$$

$$P \to p \equiv P - \frac{1}{3} P_{EW}^C \quad , \quad S \to s \equiv S - \frac{1}{3} P_{EW} \quad , \tag{1}$$

where P_{EW} and P_{EW}^C are color-favored and color-suppressed electroweak penguin amplitudes.

We use the phase conventions of Ref. [24] for pseudoscalar mesons, the mixing assumption $\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$ and $\eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}$, and the corresponding phase conventions for vector mesons with $\rho = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$, and $\phi = s\bar{s}$. We denote strangeness-preserving ($\Delta S = 0$) amplitudes by unprimed letters and strangeness-changing ($|\Delta S| = 1$) amplitudes by primed letters. The suffix on each amplitude denotes whether the spectator quark is included in a pseudoscalar (P) or vector (V) meson. (For some additional processes not listed in Table I, see Ref. [2]. No experimental results have been quoted for those processes.)

A process of the form $B \to X$, where the charge of B is not specified, will refer to both B^+ and B^0. Unless explicitly stated otherwise, we will always take the branching ratio for a process to refer to the average of that process and its charge-conjugate.

III. PATTERNS OF DOMINANT AMPLITUDES

Using the observed branching ratios quoted in Table I, we can identify reduced amplitudes for which there exists evidence. We shall assume the lifetimes of B^0 and B^+ are equal (valid to a few percent), and shall quote squares of amplitudes in units of branching ratios with a common factor of 10^{-6}. Thus, an amplitude of 1 will correspond to a branching ratio of 10^{-6}. We make qualitative estimates here, reserving more precise ones for Sec. IV.

Table I: B decay modes with contributing amplitudes and experimental branching ratios or 90% c.l. upper limits (from Ref. [3] unless stated otherwise).

Decay mode	Amplitudes	Branching ratio (units of 10^{-6}) Value (σ)	Upper limit
	B^+ Decays		
$\rho^+\pi^0$	$(-t_P + p_V - p_P - c_V)/\sqrt{2}$		77 (a)
$\rho^0\pi^+$	$(-t_V + p_P - p_V - c_P)/\sqrt{2}$	$15 \pm 5 \pm 4$ (5.2σ)	
$\omega\pi^+$	$(t_V + p_P + p_V + c_P + 2s_P)/\sqrt{2}$	$11.3^{+3.3}_{-2.9} \pm 1.5$ (6.2σ)	17
$\phi\pi^+$	s_P		4
$\rho^+\eta$	$-(t_P + p_P + p_V + c_V + s_V)/\sqrt{3}$	$4.3^{+4.3}_{-3.4} \pm 0.7$ (1.3σ)	16
$\rho^+\eta'$	$(t_P + p_P + p_V + c_V + 4s_V)/\sqrt{6}$		47
ρ^+K^0	p'_V		48 (a)
ρ^0K^+	$-(p'_V + t'_V + c'_P)/\sqrt{2}$		22
ωK^+	$(p'_V + t'_V + c'_P + 2s'_P)/\sqrt{2}$	$3.2^{+2.4}_{-1.9} \pm 0.8$ (2.1σ)	8
ϕK^+	$p'_P + s'_P$	$1.6^{+1.9}_{-1.2} \pm 0.2$ (1.3σ)	5.9
$K^{*0}\pi^+$	p'_P		27
$K^{*+}\pi^0$	$-(p'_P + t'_P + c'_V)/\sqrt{2}$		99 (a)
$\bar{K}^{*0}K^+$	p_P		12
$K^{*+}\eta$	$(p'_V - p'_P - t'_P - c'_V - s'_V)/\sqrt{3}$	$27.3^{+9.6}_{-8.2} \pm 5.0$ (4.8σ)	
$K^{*+}\eta'$	$(2p'_V + p'_P + t'_P + c'_V + 4s'_V)/\sqrt{6}$		87
	B^0 Decays		
$\rho^-\pi^+$	$-t_V - p_V$	(b)	
$\rho^+\pi^-$	$-t_P - p_P$	(b)	
$\rho^0\pi^0$	$(p_P + p_V - c_P - c_V)/2$		5.1
$\omega\pi^0$	$(p_P + p_V + c_P - c_V + 2s_P)/2$		5.8
$\phi\pi^0$	$s_P/\sqrt{2}$		5.4
$\rho^0\eta$	$-(p_P + p_V + c_V - c_P + s_V)/\sqrt{6}$	$2.6^{+3.0}_{-2.4} \pm 0.3$ (1.3σ)	11
$\rho^0\eta'$	$(p_P + p_V + c_V - c_P + 4s_V)/(2\sqrt{3})$		23
ρ^-K^+	$-p'_V - t'_V$		25
ρ^0K^0	$(p'_V - c'_P)/\sqrt{2}$		27
ωK^0	$(p'_V + c'_P + 2s'_P)/\sqrt{2}$	$10.0^{+5.4}_{-4.2} \pm 1.5$ (3.9σ)	21
ϕK^0	$p'_P + s'_P$	$10.7^{+7.8}_{-5.7} \pm 1.1$ (2.6σ)	28
$K^{*+}\pi^-$	$-p'_P - t'_P$	22^{+8+4}_{-6-5} (5.9σ)	
$K^{*0}\pi^0$	$(p'_P - c'_V)/\sqrt{2}$		4.2
$K^{*0}\eta$	$(p'_V - p'_P - c'_V - s'_V)/\sqrt{3}$	$13.8^{+5.5}_{-4.4} \pm 1.7$ (5.1σ)	
$K^{*0}\eta'$	$(2p'_V + p'_P + c'_V + 4s'_V)/\sqrt{6}$		20
$K^{*+}K^-$	(c)		6

(a) From Ref. [22].
(b) Sum of $\rho^-\pi^+$ and $\rho^+\pi^-$ is $35^{+11}_{-10} \pm 5$ (5.6σ).
(c) Contributions of order f_B/m_B or rescattering effects.

A. Evidence for t_V and t_P

The decays $B^+ \to \rho^0\pi^+$ and $B^+ \to \omega\pi^+$, with branching ratios of order 10^{-5}, are expected to be dominated by the tree amplitude t_V. The penguin amplitudes p_P and p_V are expected to be smaller than the corresponding strangeness-changing amplitudes p'_P and p'_V by roughly a factor of $|V_{td}/V_{ts}| \simeq \lambda$, where $\lambda \simeq 0.22$ is the parameter introduced by Wolfenstein [28] to describe the hierarchy of CKM matrix elements. The amplitudes p'_P and p'_V, as will be seen below, dominate processes whose branching ratios are of order 10^{-5}, so one can conclude that $(|p_V|, |p_P|) = \mathcal{O}(\lambda) \times |t_V|$. The amplitude s_P, which contributes to $B^+ \to \omega\pi^+$ but not to $B^+ \to \rho^0\pi^+$, involves an ω connected to the rest of the diagram by at least three gluons. It is expected to be small by virtue of the Okubo-Zweig-Iizuka (OZI) rule, which holds well for vector mesons. Both it and the related amplitude s'_P will be neglected, except for an electroweak penguin contribution to s'_P which will be studied in Sec. IV A. However, S'_V need not be small, since it involves flavor-singlet couplings to pseudoscalar mesons for which the OZI rule is less well satisfied. A related amplitude S' was found important [29] for the decays $B \to K\eta'$. We shall take account of S'_V when describing the decays $B \to K^*\eta$ and $B \to K^*\eta'$. The corresponding $\Delta S = 0$ amplitude S_V plays a role in $B \to \rho\eta$ and $B \to \rho\eta'$.

We conclude from Table I that $|t_V|^2/2 = \mathcal{O}(12)$ (to be multiplied, as mentioned above, by 10^{-6} to obtain the corresponding branching ratios for $B^+ \to \rho^0\pi^+$ and $B^+ \to \omega\pi^+$). The possibility that these two branching ratios differ from one another (as suggested, for example, in Ref. [21]) will be noted in Sec. IV when we come to discuss interfering subdominant amplitudes.

The decay $B^0 \to \rho^-\pi^+$ is also expected to be dominated by t_V. In the absence of separate branching ratios for this process and for $B^0 \to \rho^+\pi^-$ (which we expect to be dominated by t_P, as will be seen presently), all that is measured is the sum

$$\mathcal{B}(B^0 \to \rho^-\pi^+) + \mathcal{B}(B^0 \to \rho^+\pi^-) = (35^{+11}_{-10} \pm 5) \times 10^{-6} \quad , \tag{2}$$

which is consistent with the contribution from $|t_V|^2$ alone, but permits an additional $|t_P|^2$ contribution. Identification of the flavor of the decaying neutral B will distinguish these two decay modes from one another. Meanwhile, we anticipate indirect arguments in the next Section that bracket $|t_P|^2$ between 6.1 and 29.

B. Evidence for p'_P

The decay $B^0 \to K^{*+}\pi^-$ has been seen at the 5.9σ level with a branching ratio of $(22^{+8+4}_{-6-5}) \times 10^{-6}$. It is expected to be dominated by the penguin amplitude p'_P. The other contributing amplitude, t'_P, should be of order $|V_{us}/V_{ud}| = \lambda/(1-\lambda^2/2)$ times t_P, and since $|t_P|^2 < 29$ we expect that $|t'_P| < 1.22$.

As we shall see in Sec. IV B, it is likely that $B^0 \to K^{*+}\pi^-$ receives some enhancement from constructive interference between p'_P and other amplitudes, notably t'_P and an electroweak penguin contribution. The branching ratio for $B^+ \to \phi K^+$, also dominated by p'_P, is less than 5.9×10^{-6}, while that for $B^0 \to \phi K^0$ is $(10.7^{+7.8}_{-5.7} \pm 1.1) \times 10^{-6}$.

These constraints suggest that $|p'_P|^2 \lesssim \mathcal{O}(10)$. For comparison, values of $|p'|^2$ around 18 characterize $B \to K\pi$ decays, as will be shown in Sec. VI. (See also [30].)

C. Evidence for p'_V

The amplitude p'_V is expected to be very small in all factorization-dependent calculations of which we are aware [15, 16, 21, 23, 31], except for one [17] which appeared after the submission of this paper. In those calculations, except the one, all the decays $B \to (\omega, \rho)K$ are highly suppressed. (See, however, [32]).

In our earlier analysis of $B \to VP$ decays [2], we concluded that $p'_V \neq 0$ on the basis of the branching ratio $\mathcal{B}(B^+ \to \omega K^+) = (15 \pm 7 \pm 3) \times 10^{-6}$ reported by the CLEO Collaboration [33]. With a larger data sample and a revised analysis, the evidence for this mode has now become considerably weaker, with a 90% c.l. upper limit of 8×10^{-6} for the branching ratio [3]. However, the decay $B^0 \to \omega K^0$ indicates that $p'_V \neq 0$, with $\mathcal{B}(B^0 \to \omega K^0) = (10.0^{+5.4}_{-4.2} \pm 1.5) \times 10^{-6}$, a 3.9σ signal. As we shall see below, it is likely that $B^+ \to \omega K^+$ receives destructively interfering contributions from smaller t'_V and electroweak penguin amplitudes.

Some time ago the CLEO Collaboration reported evidence for $B^{+,0} \to K^{+,0}\eta'$ [34] at a substantial rate, supported by the data sample now analyzed [4]: $\mathcal{B}(B^+ \to K^+\eta') = (80^{+10}_{-9} \pm 8) \times 10^{-6}$, $\mathcal{B}(B^0 \to K^0\eta') = (88^{+18}_{-16} \pm 9) \times 10^{-6}$. Our interpretation of this result [29] relies in part on a large flavor-singlet amplitude s' which was proposed previously to the discovery of these processes [35]. However, an additional feature contributing to η' production through the penguin amplitude p' is constructive interference between contributions from nonstrange and strange quarks in the η', as originally proposed by Lipkin [36].

The CLEO Collaboration has now found evidence for $B^{+,0} \to K^{*(+,0)}\eta$ [4]. Lipkin argues that the same mechanism favoring $B \to K\eta'$ (with $B \to K\eta$ not yet detected) should favor $B \to K^*\eta$, with constructive interference between p'_P and p'_V in those decays and destructive interference in $B \to K^*\eta'$. His argument is equivalent to the assumption $p'_V = -p'_P$. A similar relation applies to certain contributions to charmed particle decays [37].

We shall find that in contrast to the model-dependent predictions based on factorization for penguin amplitudes, but in accord with Lipkin's suggestion, there is some evidence for the relation $p'_V \simeq -p'_P$, with $|p'_P|^2 \simeq |p'_V|^2$. The conclusion $p'_V \simeq -p'_P$ then entails $p_V \simeq -p_P$, causing the penguin contributions to many amplitudes in Table I to cancel one another.

IV. INCLUSION OF SUBDOMINANT AMPLITUDES

We summarize in Table II the evidence for interfering amplitudes to be discussed in the present Section. While much of the evidence is not yet statistically compelling, increased statistics that will be available from the CLEO III, BaBar, and Belle detectors should be able to confirm or refute the trends that are suggested by present data. By taking account of subdominant contributions, we can arrive at more accurate estimates of those amplitudes mentioned in Section III.

Table II: Patterns in $B \to VP$ data suggesting interference between dominant and subdominant amplitudes. All inequalities are those following from small final-state phases and either $\cos\gamma < 0$ ($|\Delta S| = 1$ processes) or $\cos\alpha > 0$ ($\Delta S = 0$ processes).

Subdominant amplitude	Interfering with	Consequence
t'_P	p'_P	$\Gamma(K^{*+}\pi^-) > \Gamma(\phi K^{+,0})$
t'_P	$p'_P - p'_V$	$\Gamma(K^{*+}\eta) > \Gamma(K^{*0}\eta)$
p_P	t_V	$\Gamma(\rho^0\pi^+) > \Gamma(\omega\pi^+)$
t'_V	p'_V	$\Gamma(\omega K^0) > \Gamma(\omega K^+)$

A. Contributions of electroweak penguins

We shall neglect the color-suppressed contributions $C'_{P,V}$ in strangeness-changing processes since they are highly suppressed relative to the dominant penguin amplitudes. The strangeness-conserving terms $C_{P,V}$ are subdominant to the color-allowed terms $T_{P,V}$. In B^+ decays these amplitudes occur in two specific combinations, $T_P + C_V$ and $T_V + C_P$, to which we will refer for simplicity as t_P and t_V, respectively. Since in neutral B decays T_P and T_V occur unaccompanied by C_V and C_P, this introduces a small uncertainty in estimating t_P and t_V from the measured $\Delta S = 0$ B^0 decay rates. We will use these measurements only to obtain an upper limit on $|t_P|$.

The amplitudes $c' = C' + P'_{EW}$ and $s' = S' - (1/3)P'_{EW}$ contain color-favored electroweak penguin amplitudes, which must be taken into account [26]. We employ calculations based on the factorization hypothesis [16, 38] to estimate their importance. This assumption can be tested once data become precise enough to specify the electroweak penguins directly.

We consider only strangeness-changing electroweak penguins, since they are approximately $1/\lambda^2 \simeq 20$ times as large in amplitude as strangeness-preserving ones. Furthermore, we consider only color-favored amplitudes, which appear only in the case of neutral-meson production. These are associated, through Eqs. (1), with the amplitudes $c'_{P,V}$ and $s'_{P,V}$, and the coefficients of their reduced matrix elements can be calculated either from Eqs. (1) and Table I or directly via the wave functions of the corresponding neutral mesons and quark charges [27].

We need estimates of the electroweak penguin amplitudes P'^P_{EW}, in which the spectator quark is incorporated into a pseudoscalar meson, and P'^V_{EW}, in which the spectator quark is incorporated into a vector meson. Strangeness-changing B decays with production of the vector mesons ρ^0, ω, and ϕ involve P'^P_{EW}, while those with production of π^0, η, and η' involve P'^V_{EW}.

The amplitude P'^P_{EW} has been estimated [38] to result in a 30% reduction in the predicted rate for $B^+ \to \phi K^+$ in comparison with the contribution from the gluonic penguin p'_P alone. From Eqs. (1) and Table I,

$$|A(B^+ \to \phi K^+)|^2 = |p'_P - \frac{1}{3}{P'}^P_{EW}|^2 = 0.7|p'_P|^2 \quad , \tag{3}$$

implying ${P'}^P_{EW} \simeq p'_P/2$ for zero relative strong phase between the electroweak and gluonic penguins. (The weak phases of the two are the same.) We shall assume this to be the case in what follows. The calculation of Ref. [16] gives, for $N_c = 2$, an electroweak penguin ${P'}^P_{EW}$ about 2/3 of that in Ref. [38], reaching the latter estimate for a higher value of N_c. (Here N_c is the effective number of quark colors in a $1/N_c$ expansion).

A more general fit to amplitudes could leave the relative strong phase between electroweak and gluonic penguin amplitudes as a free parameter. The relative strength of electroweak and gluonic penguin amplitudes can be tested by relating $B \to \phi K$ (which involves both) to $B^+ \to K^{*0}\pi^+$ (which involves just p'_P). With the magnitude of ${P'}^P_{EW}$ estimated in Ref. [38], the corresponding nonstrange contribution P^P_{EW} results in a predicted branching ratio $\mathcal{B}(B^+ \to \phi\pi^+) \simeq 10^{-8}$. This is completely consistent with our ansatz ${P'}^P_{EW} = p'_P/2$ if $|p'_P|^2 \simeq 10$ as noted in Sec. III and if $|P^P_{EW}| \simeq \lambda|{P'}^P_{EW}| \simeq |p'_P/10|$.

The decay $B^+ \to \rho^+ K^0$ is pure p'_V, while $B^0 \to \rho^0 K^0$ involves interference between p'_V and ${P'}^P_{EW}$. As follows from the expectation that $p'_V \simeq -p'_P$ (see Subsection IV C), this interference is expected to be constructive, resulting in more than a two-fold enhancement of $\mathcal{B}(B^0 \to \rho^0 K^0)$ relative to the prediction in the absence of the electroweak penguin. The comparison of rates for these two processes thus is a way to learn ${P'}^P_{EW}/p'_V$. Assuming that these amplitudes are relatively real, we predict that

$$\frac{2\Gamma(\rho^0 K^0)}{\Gamma(\rho^+ K^0)} = \left(1 - \frac{{P'}^P_{EW}}{p'_V}\right)^2 > 1 \quad . \tag{4}$$

The amplitude ${P'}^V_{EW}$ is estimated in [16] to contribute to $B^+ \to K^{*+}\pi^0$ in such a way that ${P'}^V_{EW}/p'_P = (0.28\text{–}0.35,\ 0.23\text{–}0.29,\ 0.14\text{–}0.20)$ for $N_c = (2,\ 3,\ \infty)$. We shall assume the nominal value ${P'}^V_{EW}/p'_P = 1/4$.

The relative phase expected between ${P'}^V_{EW}$ and p'_P in Refs. [38] is consistent with that in [16]. Both find destructive interference in $B^0 \to K^{*0}\pi^0$. By comparing the rates for $B^+ \to K^{*0}\pi^+$ (pure p'_P) and $B^0 \to K^{*0}\pi^0$ (with gluonic-electroweak penguin interference), one should be able to deduce ${P'}^V_{EW}/p'_P$ from experiment. Assuming that the two amplitudes are relatively real, we predict that

$$\frac{2\Gamma(K^{*0}\pi^0)}{\Gamma(K^{*0}\pi^+)} = \left(1 - \frac{{P'}^V_{EW}}{p'_P}\right)^2 < 1 \quad . \tag{5}$$

We summarize the expectations for EWP contributions in Table III. We quote estimates of ratios with respect to gluonic penguin amplitudes since they are probably more reliable than those of absolute magnitudes. We will see that the contributions in Table III lead to a self-consistent picture. Eventually it will be possible to determine the electroweak penguin contributions themselves from the data. Predicted inequalities based on electroweak penguin contributions are summarized in Table IV.

Table III: Color-favored electroweak penguin contributions to $|\Delta S| = 1$ B decays.

Decay Mode	P'^{P}_{EW} Coeff.	Contrib.	Decay Mode	P'^{V}_{EW} Coeff.	Contrib.
$\rho^0 K$	$-\frac{1}{\sqrt{2}}$	$-0.35p'_P$	$K^*\pi^0$	$-\frac{1}{\sqrt{2}}$	$-0.18p'_P$
ωK	$\frac{1}{3\sqrt{2}}$	$0.12p'_P$	$K^*\eta$	$-\frac{2}{3\sqrt{3}}$	$-0.10p'_P$
ϕK	$-\frac{1}{3}$	$-0.17p'_P$	$K^*\eta'$	$-\frac{1}{3\sqrt{6}}$	$-0.03p'_P$

Table IV: Predicted inequalities due to interference between electroweak and gluonic penguin amplitudes.

Subdominant amplitude	Interfering with	Consequence
P'^{P}_{EW}	p'_P	$\Gamma(\phi K^+) = \Gamma(\phi K^0) < \Gamma(K^{*0}\pi^+)$
P'^{P}_{EW}	p'_V	$2\Gamma(\omega K^0) < \Gamma(\rho^+ K^0) < 2\Gamma(\rho^0 K^0)$
P'^{V}_{EW}	p'_P	$\Gamma(K^{*0}\pi^+) > 2\Gamma(K^{*0}\pi^0)$

B. Evidence for t'_P–p'_P interference from $B^0 \to K^{*+}\pi^-$ and $B \to \phi K$

We present in this subsection the main evidence for negative $\cos\gamma$, through the enhancement of the decay $B^0 \to K^{*+}\pi^-$ relative to its contribution from the penguin amplitude alone. Including the contribution of the electroweak penguin amplitude, we noted in the previous subsection that [38] $|A(B \to \phi K)|^2 = 0.7|p'_P|^2$. Using the experimental upper limit [3] $\mathcal{B}(B^+ \to \phi K^+) < 5.9\times10^{-6}$, we then estimate $|p'_P|^2 < 8.4$ or $|p'_P| < 2.90$ (90% c.l. as usual). The branching ratio for $B^0 \to \phi K^0$ is consistent with this bound.

On the other hand, $\mathcal{B}(B^0 \to K^{*+}\pi^-) > 12\times10^{-6}$ at 90% c.l., implying $\overline{|p'_P + t'_P|^2} > 12$ for the charge-averaged rate. We also have the constraint $|t'_P| < 1.22$ which was mentioned in Sec. III B. The weak phase of p'_P is π and the weak phase of t'_P is γ. We temporarily relax our assumption of vanishing strong phases, assuming only that the relative strong phase δ between p'_P and t'_P satisfies $\cos\delta > 0$. This assumption appears reasonable on the basis of both perturbative [39, 40] and statistical [41] estimates. We then find that the equations

$$|p'_P| < 2.90 \quad , \quad |t'_P| < 1.22 \quad ,$$

$$\overline{|p'_P + t'_P|^2} = |p'_P|^2 + |t'_P|^2 - 2|p'_P t'_P| \cos\gamma \cos\delta > 12 \tag{6}$$

have a solution only for the range

$$2.25 < |p'_P| < 2.90 \quad , \quad 0.56 < |t'_P| < 1.22 \quad , \quad 107^\circ < \gamma \le 180^\circ \quad , \tag{7}$$

where the value $\gamma = 107^\circ$ corresponds to the maximum values $|p'_P| = 2.90$, $|t'_P| = 1.22$, $\cos\delta = 1$. Thus, constructive interference between t'_P and p'_P, corresponding to

$\cos\gamma < 0$, is required. The value $\gamma = 107°$ is marginally consistent with the range specified in Ref. [19], but far outside the aggressive limits quoted in Ref. [20].

C. Information on p'_V and S'_V from $B \to (\omega K, K^*\eta, K^*\eta')$

We define the parameters κ and μ by $S'_V = \kappa p'_P$, $p'_V = -\mu p'_P$. We assume κ and μ are real, thereby neglecting a possible strong phase. A set of constraints on κ and μ will be deduced from bounds on the magnitudes of the following amplitudes:

$$A(B^0 \to \omega K^0) = [p'_V + \frac{1}{3}{P'}^P_{EW}]/\sqrt{2} = (-\mu + \frac{1}{6})p'_P/\sqrt{2} \quad , \tag{8}$$

$$A(B^+ \to K^{*+}\eta) = [p'_V - p'_P - t'_P - \frac{2}{3}{P'}^V_{EW} - S'_V]/\sqrt{3} = -[(\mu+\kappa+\frac{7}{6})p'_P + t'_P]/\sqrt{3} \ , \tag{9}$$

$$A(B^0 \to K^{*0}\eta) = [p'_V - p'_P - \frac{2}{3}{P'}^V_{EW} - S'_V]/\sqrt{3} = -(\mu + \kappa + \frac{7}{6})p'_P/\sqrt{3} \quad , \tag{10}$$

$$A(B^0 \to K^{*0}\eta') = [2p'_V + p'_P - \frac{1}{3}{P'}^V_{EW} + 4S'_V]/\sqrt{6} = (-2\mu + 4\kappa + \frac{11}{12})p'_P)\sqrt{6} \quad . \tag{11}$$

The 90% c.l. bounds we use, based on the data in Table I, are

$$4.3 < |A(\omega K^0)|^2 < 17.2 \ , \quad 15.0 < |A(K^{*+}\eta)|^2 \quad ,$$
$$|A(K^{*0}\eta)|^2 < 21.2 \ , \quad |A(K^{*0}\eta')|^2 < 20 \quad . \tag{12}$$

Other experimental bounds turn out to give weaker conditions on κ and μ. Taking account of the maximum magnitude $|t'_P| < 1.22$, these bounds lead to the conditions

$$2.93 < \left|\mu - \frac{1}{6}\right| |p'_P| < 5.86 \ , \quad 5.49 < \left|\mu + \kappa + \frac{7}{6}\right| |p'_P| < 7.97 \quad ,$$
$$\left|4\kappa - 2\mu + \frac{11}{12}\right| |p'_P| < 10.95 \quad . \tag{13}$$

With $|p'_P| = 2.90$, these relations then have a solution within the trapezoidal region bounded by the points $(\kappa, \mu) = (-0.45, 1.18)$, $(0.40, 1.18)$, $(-0.26, 1.84)$, and $(-0.54, 1.26)$. With $|p'_P| = 2.25$ the corresponding region is bounded by $(-0.20, 1.47)$, $(0.91, 1.47)$, $(-0.17, 2.55)$, and $(-0.54, 1.81)$. Negative values of μ violate the last constraint in Eq. (13), leading to unacceptably high values of the branching ratio for $B^0 \to K^{*0}\eta'$.

The bound $\mu > 1.18$ is very close to the situation in Ref. [36], where $p'_V = -p'_P$, and very far from the model calculations in which $|p'_V| \ll |p'_P|$. Thus, we will assume in what follows that $p'_V = -p'_P$ or $\mu = 1$, and will tolerate a small discrepancy ($\sim 1.6\sigma$) with respect to the observed value of $\mathcal{B}(B^0 \to \omega K^0)$.

For $\mu = 1$ and $|p'_P| = 2.90$, the value of $\kappa = S'_V/p'_P$ is bounded between -0.28 and 0.58 by the second of Eqs. (13). The corresponding range for $|p'_P| = 2.25$ is $0.27 < \kappa < 1.38$. We shall take $\kappa = 0.58$. In Sec. VI A we shall show that a similar ratio is characteristic of S'/P' in $B \to PP$ decays. As in Ref. [29], there is still some uncertainty in the contribution of the flavor-singlet amplitude which must be resolved with the help of better data on decays involving η and η'.

With these parameters, with $|p'_p| < 2.90$, and with maximally constructive interference, one finds $\mathcal{B}(B^+ \to K^{*+}\eta) < 28 \times 10^{-6}$. With no interference, the maximum branching ratio is about 22×10^{-6}. Thus, in order to demonstrate convincing interference between t'_P and the remaining amplitudes in $B^+ \to K^{*+}\eta$, it will probably be necessary to measure the difference in branching ratios for $B^{(+,0)} \to K^{*(+,0)}\eta$ to about 10%.

The large value of $|p'_V| > 2.25$ from $B^0 \to \omega K^0$ stands in sharp contradiction to most explicit models. It implies that $\mathcal{B}(B^+ \to \rho^+ K^0) > 5 \times 10^{-6}$. In contrast, for example, Ref. [21] predicts $\mathcal{B}(B^+ \to \rho^+ K^0) = 6 \times 10^{-7}$, nearly an order of magnitude lower.

With $p'_V = -p'_P$, $2.25 < |p'_P| < 2.90$, and $S'_V = 0.58 p'_P$, the branching ratio for $B^0 \to K^{*0}\eta$ is predicted to range between 13 and 21×10^{-6}, in satisfactory agreement with the experimental value.

D. Information on t_V and t_P from $B^+ \to (\rho^0, \omega)\pi^+$ and $B^0 \to \rho^\mp \pi^\pm$ rates

As mentioned in Sec. III A, the dominant amplitude contributing to $B^+ \to \rho^0\pi^+$ and $B^+ \to \omega\pi^+$ is t_V. The central values of the branching ratios quoted in Table I correspond to differences of tens of percent between these two rates. This trend is what one expects if the dominant amplitude t_V interferes with p_P constructively in $\rho^0\pi^+$ and destructively in $\omega\pi^+$. From the bounds on p'_P obtained earlier and the expectation $|p_P| \simeq \lambda |p'_P|$, one finds $0.49 < |p_P| < 0.64$ (90% c.l.). The relative phase between p_P and t_V should be $\beta + \gamma = \pi - \alpha$ in the limit that one neglects up and charmed quark contributions [42] to p_P. Thus, for $\cos\alpha > 0$, one will get constructive interference between t_V and p_P in $\rho^0\pi^+$ and destructive interference in $\omega\pi^+$.

We should mention the importance of the up and charmed quark contributions to $\bar{b} \to \bar{d}$ penguin amplitudes noted in Ref. [42]. In the limit in which the charm contribution dominates, one measures $-\cos\gamma$ instead of $\cos\alpha$. Since $\gamma = \pi - \alpha - \beta$ and β is limited to a small range around 20°, the values of $\cos\alpha$ and $-\cos\gamma$ are not all that different. However, without an explicit evaluation of the relative up, charm, and top contributions to the $\bar{b} \to \bar{d}$ penguins, the interpretation of tree-penguin interference in $\Delta S = 0$ processes is open to some question [43].

The coefficient of p_V is of the same sign as that of t_V in $B^+ \to \rho^0\pi^+$ and $B^+ \to \omega\pi^+$. With the conjectured relation $p_V = -p_P$ mentioned at the end of Section III, one then has

$$A(B^+ \to \rho^0\pi^+) = (-t_V + 2p_P)/\sqrt{2} \quad , \quad A(B^+ \to \omega\pi^+) = t_V/\sqrt{2} \quad , \tag{14}$$

so that one may use the latter process to estimate $|t_V|^2$, with the result

$$2|A(B^+ \to \omega\pi^+)|^2 = |t_V|^2 = 22.6^{+7.3}_{-6.5} \quad , \tag{15}$$

or $3.78 < |t_V| < 5.65$, $0.85 < |t'_V| = |V_{us}/V_{ud}||t_V| < 1.28$ at 90% c.l. Assuming top-quark dominance of p_P, the phase α can then in principle be deduced from the rate for $B^+ \to \rho^0\pi^+$. With present data (neglecting strong phase differences to display maximal interference effects), the rate

$$2|A(B^+ \to \rho^0\pi^+)|^2 = ||t_V| + 2e^{i\alpha}|p_P||^2 = 30 \pm 13 \quad , \tag{16}$$

still permits all values of $\cos\alpha$. However, if we were to take the central value of the $\omega\pi^+$ branching ratio seriously, we would obtain $B(\rho^0\pi^+) = (17, 12) \times 10^{-6}$ for (fully constructive, no) interference between t_V and $p_V - p_P = -2p_P$. As in the case of the $K^*\eta$ decays mentioned above, a 10% measurement of the $\omega\pi^+$ and $\rho\pi^+$ branching ratios would be needed to begin to shed light on the expected magnitude of the interference term.

We now use data on $B^0 \to \rho^\mp\pi^\pm$ to specify the upper bound (mentioned in Sec. III A) on $|t_P|$. We use the fact that in $|A(\rho^-\pi^+)|^2 = |t_V + p_V|^2$ and $|A(\rho^+\pi^-)|^2 = |t_P + p_P|^2$, the relations $p_V \simeq -p_P$ and $t_V \simeq t_P$ cause the interference terms to cancel approximately, so that the sum of the rates yields

$$|t_V|^2 + |t_P|^2 + |p_V|^2 + |p_P|^2 \simeq 35^{+12.1}_{-11.2} \quad . \tag{17}$$

Taking $0.49 < |p_V| \simeq |p_P| < 0.64$ and $|t_V|^2$ as specified above, we find $|t_P|^2 < 29$ at 90% c.l. The corresponding upper bound on $|t'_P|$ is 1.22, as mentioned in Sec. III B. The lower bound on $|t'_P|$ implies $|t_P| > 2.48$, $|t_P|^2 > 6.1$.

E. Effect of t'_V in ωK decays

Taking account of electroweak penguin contributions, the amplitudes for ωK decays are dominated by the terms

$$A(B^+ \to \omega K^+) = (p'_V + t'_V + 0.17p'_P)/\sqrt{2} \quad , \tag{18}$$

$$A(B^0 \to \omega K^0) = (p'_V + 0.17p'_P)/\sqrt{2} \quad . \tag{19}$$

With $p'_V = -p'_P$ (as chosen in our analysis of $K^*(\omega, \eta)$ decays), we then find

$$A(B^+ \to \omega K^+) = (t'_V - 0.83p'_P)/\sqrt{2} \quad , \quad A(B^0 \to \omega K^0) = -0.83p'_P/\sqrt{2} \quad . \tag{20}$$

The last equation implies $1.8 \times 10^{-6} < B(B^0 \to \omega K^0) < 2.9 \times 10^{-6}$ for $2.25 < |p'_P| < 2.90$. The experimental branching ratio is compatible with this range at 1.6σ (95% c.l.), as mentioned above. We note in passing that we predict $\Gamma(B^0 \to \omega K^0) = \Gamma(B^+ \to \phi K^+)/2 = \Gamma(B^0 \to \phi K^0)/2$. Since $B^0 \to \omega K^0$ has been seen while $B \to \phi K$ has not (the ϕK^0 signal is only 2.6σ), this could be the first test capable of falsifying our assumption that $p'_V = -p'_P$.

Since the weak phase of t'_V is γ and that of p'_P is π, we get destructive interference between t'_V and p'_P in the charge-averaged $\omega K^\pm$ rate (when $\cos\delta > 0$) for $\cos\gamma < 0$. Taking $0.85 < |t'_V| < 1.28$ as suggested above, $|p'_P| = 2.90$, and maximally destructive interference in $B^+ \to \omega K^+$, we get $B(B^+ \to \omega K^+) = 10^{-6}$. Thus, although we predict p'_V to have a considerably larger magnitude than do many specific models, its effect in $B^+ \to \omega K^+$ can be considerably diminished by destructive interference with both tree and electroweak penguin contributions.

V. SUMMARY OF PREDICTIONS FOR $B \to VP$ DECAYS

We have deduced ranges for the main amplitudes governing $B \to VP$ decays from existing data and bounds. Within these ranges, some of the penguin amplitudes (p_V,

Table V: Summary of 90% c.l. bounds or assumed values for amplitudes contributing to $B \to VP$ decays.

$\Delta S = 0$		$\lvert\Delta S\rvert = 1$	
Amplitude	Range	Amplitude	Range
t_V	3.78–5.65	t'_V	0.85–1.28
t_P	2.48–5.39	t'_P	0.56–1.22
p_P	0.49–0.64	p'_P	2.25–2.90
p_V	$-p_P$	p'_V	$-p'_P$
S_V	$0.58p_P$	S'_V	$0.58p'_P$
P_{EW}^P	$p_P/2$	${P'}_{EW}^P$	$p'_P/2$
P_{EW}^V	$p_P/4$	${P'}_{EW}^V$	$p'_P/4$

S_V, $P_{EW}^{P,V}$ and the corresponding $|\Delta S| = 1$ amplitudes) were chosen to have specific ratios relative to p_P or p'_P. Our results (at 90% c.l.) are summarized in Table V. They imply that many of the processes listed in Table I should be observable at branching ratio levels of a few parts in 10^6. These are summarized in Tables VI and VII. Some were already noted in the previous Sections. In addition to predictions for the sign of interference with $\cos\gamma < 0$ and $\cos\alpha > 0$, shown by numbers in parentheses and mentioned in the text, predictions for the opposite sign of interference with $\cos\gamma > 0$ and $\cos\alpha < 0$ are shown in brackets.

A. $\rho\pi$ decays

The decay $B^+ \to \rho^+\pi^0$ should be dominated by $|t_P|$, whose square we estimated to be between 6.1 and 29 (in our usual units), leading to a prediction $\mathcal{B}(B^+ \to \rho^+\pi^0)$ between 3.6 and 15×10^{-6} in the absence of tree-penguin interference. If interference between t_P and $p_P - p_V \simeq 2p_P$ occurs, it is likely to be destructive for $\cos\alpha > 0$, and could reduce the lower bound to less than 10^{-6}. In $B^0 \to \rho^-\pi^+$ we expect

$$A(B^0 \to \rho^-\pi^+) = -(t_V + p_V) \simeq -(t_V - p_P) \quad , \tag{21}$$

with $B(B^0 \to \rho^-\pi^+)$ ranging from 15×10^{-6} (for the smallest acceptable $|t_V|$ and no interference) to 40×10^{-6} (for the largest acceptable $|t_V|$ and constructive interference, as expected if $\cos\alpha > 0$). With $A(B^0 \to \rho^+\pi^-) = -(t_P + p_P)$ we expect $B(B^0 \to \rho^+\pi^-)$ ranging from 6.3 to 29×10^{-6} (for no interference) or as low as 3.3×10^{-6} (with destructive interference, as expected if $\cos\alpha > 0$).

B. $\rho^+\eta$ and $\rho^+\eta'$ decays

The decays $B^+ \to \rho^+\eta$ and $B^+ \to \rho^+\eta'$ should be characterized by branching ratios ranging from 2 to 11×10^{-6} and 0.2 to 8×10^{-6}, respectively. Some events of $\rho^+\eta$ have already been observed [3], but the significance of the signal is marginal.

Table VI: Predicted 90% c.l. ranges for $B^+ \to VP$ branching ratios in units of 10^{-6}. Values are quoted for no tree-penguin interference. Those in parentheses denote possible values when such interference is present with $-1 < \cos\gamma < 0$ or $0 < \cos\alpha < 1$, while those in brackets denote corresponding possible values with $0 < \cos\gamma < 1$ or $-1 < \cos\alpha < 0$. Charge-averaged branching ratios are understood here.

Mode	Predicted b.r.
$\rho^+\pi^0$	3.6 (0.7)–15 [22]
$\rho^0\pi^+$	7.8 [3.0]–17 (24)
$\omega\pi^+$	7.1–16 (a)
$\rho^+\eta$	2.1 (1.5)–9.7 [11]
$\rho^+\eta'$	1.2 (0.2)–5.2 [7.9]
ρ^+K^0	5–8.4
ρ^0K^+	1.0 (0)–1.9 [3.7]
ωK^+	2.1 (0.2)–3.8 [6.8]
ϕK^+	3.5–5.9 (b)
$K^{*0}\pi^+$	5–8.4
$K^{*+}\pi^0$	4.1 [1.3]–7.3 (12)
$K^{*+}\eta$	13 [8.2]–22 (28)
$K^{*+}\eta'$	1.3 [0.4]–2.4 (3.8)

(a) Input value was $11.3^{+3.6}_{-3.3}$
(b) Upper bound served as input

Table VII: Predicted ranges for $B^0 \to VP$ branching ratios. (See caption to Table VI for details.)

Mode	Predicted b.r.
$\rho^-\pi^+$	15 [11]–32 (40)
$\rho^+\pi^-$	6.3 (3.3)–29 [37]
ρ^-K^+	5.6 (0.9)–10 [17]
ρ^0K^0	5.7–9.5
ωK^0	1.8–2.9
ϕK^0	3.5–5.9
$K^{*+}\pi^-$	5.4 [1.1]–9.9 (17) (a)
$K^{*0}\pi^0$	1.4–2.4
$K^{*0}\eta$	13–21 (b)
$K^{*0}\eta'$	1.3–2.4

(a) Input value was 22^{+9}_{-8}
(b) Upper bound served as input

This process may be a useful one to measure the magnitude of t_P, since the (small) penguin contributions p_V and p_P are expected to cancel one another approximately.

C. ρK decays

The decay $B^+ \to \rho^+ K^0$ is due entirely to the p'_V amplitude. Thus we expect $\mathcal{B}(B^+ \to \rho^+ K^0)$ to range between 5 and 8.4×10^{-6}. With $|p_V|^2 \simeq |p'_V|^2/20$ we then expect $\mathcal{B}(B^+ \to K^{*+}\bar{K}^0)$ and $\mathcal{B}(B^0 \to K^{*0}\bar{K}^0)$ to range between 0.3 and 0.4×10^{-6}. At these low levels it is hard to exclude the possibility that rescattering effects could feed these channels from other more abundant ones.

The decay $B^+ \to \rho^0 K^+$ should have a branching ratio very similar to that for $B^+ \to \omega K^+$ except for the electroweak penguin contribution. However, this contribution is expected to be appreciable, further reducing the magnitude of the amplitude through destructive interference with p'_V and t'_V:

$$A(B^+ \to \rho^0 K^+) = -(p'_V + t'_V + 0.5p'_P)/\sqrt{2} = -(t'_V - 0.5p'_P)/\sqrt{2} \quad , \tag{22}$$

leading to a branching ratio of 1 to 1.9×10^{-6} if there is no interference between t'_V and p'_P and possibly even less if there is destructive interference as expected for $\cos\gamma < 0$.

The decay $B^0 \to \rho^- K^+$ has the same matrix element as $B^+ \to \rho^0 K^+$ except that it is missing the electroweak penguin contribution and is a factor of $\sqrt{2}$ larger:

$$A(B^+ \to \rho^- K^+) = -(p'_V + t'_V) \quad . \tag{23}$$

With no interference, the expected range of branching ratios is about 5.6 to 10×10^{-6}, while destructive interference for $\cos\gamma < 0$ could push the branching ratio below 10^{-6}.

The decay $B^0 \to \rho^0 K^0$ is expected to exhibit constructive interference between the gluonic and electroweak penguin amplitudes:

$$A(B^0 \to \rho^0 K^0) = \frac{p'_V - c'_P}{\sqrt{2}} \simeq \frac{p'_V - P'^P_{EW}}{\sqrt{2}} \simeq \frac{-p'_P - 0.5p'_P}{\sqrt{2}} \quad , \tag{24}$$

leading to a predicted range for the branching ratio between 5.7 and 9.5×10^{-6}, an order of magnitude greater than that in the model of Ref. [15].

D. ϕK decays

The anticipated limits on $|p'_P|$ lead to $\mathcal{B}(B^+ \to \phi K^+)$ between 3.5 and 5.9×10^{-6}. Suppression below this range would cast doubt most likely on our assumption of flavor SU(3), which required the amplitude for $s\bar{s}$ production in the final state to be the same as that for $u\bar{u}$ or $d\bar{d}$ production. Another possibility is that electroweak penguin contributions lead to a stronger suppression than in Eq. (3). We expect $\mathcal{B}(B^+ \to \phi K^+) = \mathcal{B}(B^0 \to \phi K^0)$, so a similar range is expected for $B^0 \to \phi K^0$.

E. $K^*\pi$ decays

The decay $B^+ \to K^{*0}\pi^+$ is expected to be dominated by the p'_P amplitude, and thus to have a range of branching ratios between 5 and 8.4×10^{-6}. The corresponding $\Delta S = 0$ processes, $B^+ \to \bar{K}^{*0}K^+$ and $B^0 \to \bar{K}^{*0}K^0$, should have branching ratios between 0.3 and 0.4×10^{-6}.

The gluonic and electroweak penguin amplitudes are the main contributors to $B^0 \to K^{*0}\pi^0$, leading to an amplitude $A = (p'_P - P'^{V}_{EW})/\sqrt{2} = 0.75p'_P/\sqrt{2}$. With $2.25 < |p'_P| < 2.9$, we then predict $1.4 \times 10^{-6} < \mathcal{B}(B^0 \to K^{*0}\pi^0) < 2.4 \times 10^{-6}$, in accord with the upper limit of 4.2×10^{-6} for this branching ratio.

In the decay $B^+ \to K^{*+}\pi^0$, constructive tree-penguin interference can occur for $\cos\gamma < 0$. If no such interference occurs, one expects the branching ratio to range between 4.1 and 7.3×10^{-6}, while it can reach as high as 12×10^{-6} if the constructive interference is present.

F. $K^*\eta$ and $K^*\eta'$ decays

The tree-penguin interference is expected to be constructive for $\cos\gamma < 0$ in $B^+ \to K^{*+}\eta$, leading to $\mathcal{B}(B^+ \to K^{*+}\eta) > \mathcal{B}(B^0 \to K^{*0}\eta)$. For the value of S'_V chosen here, it is also expected to be constructive for $\cos\gamma < 0$ in $B^+ \to K^{*+}\eta'$, leading to $\mathcal{B}(B^+ \to K^{*+}\eta') > \mathcal{B}(B^0 \to K^{*0}\eta')$. The exact magnitude of these branching ratios is very sensitive to S'_V. The large disparity between $K^*\eta$ and $K^*\eta'$ is only possible when p'_V has an appreciable magnitude, comparable to that of p'_P, in contrast to the estimates based on factorization.

G. Other processes with neutral mesons

The processes $B^0 \to (\rho^0, \omega) + (\pi^0, \eta, \eta')$ are expected to have very small rates. The gluonic penguin contributions p_P and p_V are predicted to cancel one another, leaving only small electroweak penguin terms. At this level we cannot exclude the possibility that color-suppressed amplitudes, neglected here, play a role, so we estimate only that these branching ratios are all less than 10^{-6}, and do not quote them in Tables VI and VII.

VI. IMPLICATIONS FOR OTHER PROCESSES

A. $B \to PP$ decays

The CLEO Collaboration [5] has presented evidence for two $B \to \pi\pi$ modes, four $B \to K\pi$ modes and two $B \to K\eta'$ modes [4], as shown in Table VIII. Also shown are decompositions into SU(3)-invariant amplitudes, including explicit contributions of color-favored electroweak penguins. (The smaller contributions of color-suppressed electroweak penguins are omitted.) We make several observations about these results.

1. *Sum rule for $K\pi$ decay rates.* Lipkin [44] has noted that the $B \to K\pi$ rates satisfy the relation

$$\Gamma(K^+\pi^-) + \Gamma(K^0\pi^+) = 2[\Gamma(K^+\pi^0) + \Gamma(K^0\pi^0)] \tag{25}$$

Table VIII: $B \to \pi\pi$, $B \to K\pi$ and $B \to K\eta'$ decay modes, with decomposition into invariant amplitudes.

Decay Mode	Amplitudes	B.r. (units of 10^{-6}) (σ)
$\pi^+\pi^-$	$-(T+P)$	$4.7^{+1.8}_{-1.5} \pm 0.6$ (4.2σ)
$\pi^+\pi^0$	$-(T+C+P_{EW})/\sqrt{2}$	$5.4^{+2.1}_{-2.0} \pm 1.5$ (3.2σ)
$K^+\pi^-$	$-(T'+P')$	$18.8^{+2.8}_{-2.6} \pm 1.3$ (11.7σ)
$K^+\pi^0$	$-(T'+P'+C'+P'_{EW})/\sqrt{2}$	$12.1^{+3.0+2.1}_{-2.8-1.4}$ (6.1σ)
$K^0\pi^+$	P'	$18.2^{+4.6}_{-4.0} \pm 1.6$ (7.6σ)
$K^0\pi^0$	$(P'-C'-P'_{EW})/\sqrt{2}$	$14.8^{+5.9+2.4}_{-5.1-3.3}$ (4.7σ)
$K^+\eta'$	$(3P'+4S'+T'+C'-(1/3)P'_{EW})/\sqrt{6}$	$80^{+10}_{-9} \pm 8$ (16.8σ)
$K^0\eta'$	$(3P'+4S'+C'-(1/3)P'_{EW})/\sqrt{6}$	$88^{+18}_{-16} \pm 9$ (11.7σ)

when dominated by the P' amplitude and expanded to leading order in smaller amplitudes. A slightly more general proof of this relation was given in Ref. [45]. With the new experimental values, this relation [in units of (branching ratio $\times 10^6$)] reads

$$37.0^{+5.8}_{-5.2} = 53.8^{+14.7}_{-13.7} \quad , \tag{26}$$

in satisfactory agreement at the $\sim 1\sigma$ level.

2. *Complete P' dominance.* In the limit in which all amplitudes except P' can be neglected one has $\Gamma(K^+\pi^-) = 2\Gamma(K^+\pi^0) = \Gamma(K^0\pi^+) = 2\Gamma(K^0\pi^0)$, which is also in satisfactory agreement with experiment. Thus, at the moment, there are no indications from $K\pi$ decays for the amplitudes T', C', or P'_{EW}.

3. *Bound on γ from $\Gamma(K^0\pi^+)/[2\Gamma(K^+\pi^0)]$.* A test for interference of subdominant amplitudes with P', taking account of electroweak penguin effects, can be based on the deviation of the ratio $R^* \equiv \Gamma(B^+ \to K^0\pi^+)/[2\Gamma(B^+ \to K^+\pi^0)]$ from unity [10, 11, 12, 13]. With present data $R^* = 0.75 \pm 0.28$, not differing significantly from 1.

4. *Bound on γ from $\Gamma(K^+\pi^-)/\Gamma(K^0\pi^+)$.* An earlier test for interference of T' with P' [8] becomes useful when the ratio $R \equiv \Gamma(B^0 \to K^+\pi^-)/\Gamma(B^+ \to K^0\pi^+)$ lies below 1: $\sin^2\gamma \le R$. Since with present data $R = 1.03 \pm 0.31$, no useful bound results. It may be possible to combine data on charge asymmetries [6] with information on R or R^* to place bounds on γ [9, 11, 12, 14, 46].

5. *Information from $B \to \pi\pi$ decays.* We may briefly update previous analyses (see, e.g., [29, 30]) of $B \to PP$ using the new results in Table VIII. The small branching ratio for $B^0 \to \pi^+\pi^-$) will be seen to favor $\cos\alpha \ge 0$, in accord with other recent claims [5, 15, 17, 18], but only at the 1σ level.

We first estimate the amplitude T from the rate for the decay $B^+ \to \pi^+\pi^0$. We find from Table VIII that $|T+C|^2/2 = 5.4 \pm 2.5$. We need an estimate of the small contribution C and assume for present purposes the validity of the calculation in Ref. [40] whereby $\mathrm{Re}(C/T) = \mathcal{O}(0.1)$. (As in estimates of electroweak penguins, we

Table IX: Summary of amplitudes contributing to $B \to PP$ decays.

$\Delta S = 0$		$\|\Delta S\| = 1$	
Amplitude	Value	Amplitude	Value
$\|T\|$	3.0 ± 0.7	$\|T'\|$	$\lambda\|T\|f_K/f_\pi$
$\|P\|$	$\lambda\|P'\| = 0.94 \pm 0.12$	$\|P'\|$	4.3 ± 0.5
S	$\lambda\|S'\|$	$\|S'\|$	$(0.6 \pm 0.2)\|P'\|$ (a)

(a) Assuming constructive interference between S' and P' in $B \to K\eta'$ decays.

place more reliance on ratios of amplitudes than on absolute magnitudes.) We then find $|T| = 3.0 \pm 0.7$, in satisfactory agreement with other estimates [47, 48].

The penguin amplitude P may be estimated from the process $B^+ \to K^0\pi^+$, which is expected to be pure penguin: $|P'|^2 = 18.2 \pm 4.6$, or $|P'| = 4.3 \pm 0.5$. Then $|P| = \lambda|P'| = 0.94 \pm 0.12$.

Assuming top-quark dominance of P [42], the weak phase α and the relative strong phase δ then are constrained by the charge-averaged $\pi^+\pi^-$ branching ratio:

$$|T|^2 + |P|^2 - 2|TP| \cos\alpha \cos\delta = 4.7 \pm 1.8 \quad , \tag{27}$$

$$0.9 \pm 0.9 = \cos\alpha \cos\delta \quad . \tag{28}$$

Thus, assuming $\cos\delta > 0$, one favors $\cos\alpha > 0$, but only at the $\sim 1\sigma$ level.

6. Ratio of singlet to penguin amplitudes S'/P'. Taking the dominant terms in the $B^0 \to K^0\eta'$ decays (thereby avoiding possible complications associated with the T' contribution to $B^+ \to K^+\eta'$),

$$A(B^0 \to K^0\eta') \simeq \frac{3P' + 4S'}{\sqrt{6}} \quad , \tag{29}$$

neglecting color-suppressed and electroweak penguin terms, we can estimate the ratio S'/P' in the case of constructive interference (the case considered for the ratio S'_V/p'_P contributing to $B \to K^*\eta$). We find $S'/P' = 0.6 \pm 0.2$, which is very close to the value taken for S'_V/p'_P.

7. Reduced amplitudes for $B \to PP$ and comparison with those for $B \to VP$. We summarize the reduced amplitudes for $B \to PP$ amplitudes found in the previous paragraphs in Table IX. (We do not include the electroweak penguin amplitudes discussed in Refs. [10]-[14].)

The value of $|T|$ found here is comparable to those found for $|t_P|$ and $|t_V|$ in Table V. One might expect (see, e.g., [49]) that $|t_P/T| \simeq f_\rho/f_\pi \simeq \sqrt{2}$, which is also consistent with present data. The value of $|P'|$ is somewhat larger than our values of $|p'_P|$ and $|p'_V|$, where the smallness of $|p'_P|$ is dictated in part by the need to accommodate a small branching ratio for $B \to \phi K$. We have already commented on the fact that $|S'_V/P'_V| \simeq |S'/P'|$ is consistent with present data.

B. $B_s - \bar{B}_s$ mixing

The comparison of the $B^0 - \bar{B}^0$ mixing parameter $\Delta m_d = 0.464 \pm 0.018$ ps^{-1} and the $B_s - \bar{B}_s$ mixing parameter $\Delta m_s > 14.3$ ps^{-1} (95% c.l.) [50] provides information on $\cos\gamma$. Defining f_B and f_{B_s} as the nonstrange and strange B meson decay constants and B_B and B_{B_s} as the corresponding vacuum-saturation factors, we have

$$\sqrt{\frac{\Delta m_s}{\Delta m_d}} = \frac{f_{B_s}\sqrt{m_{B_s}B_{B_s}}}{f_B\sqrt{m_B B_B}}\left|\frac{V_{ts}}{V_{td}}\right| \quad , \tag{30}$$

leading, with $f_{B_s}\sqrt{B_{B_s}}/[f_B\sqrt{B_B}] < 1.25$ [49], to $|V_{ts}/V_{td}| > 4.32$ and $|V_{td}/A\lambda^3| = |1-\rho-i\eta| < 1.05$. Thus only a small region of $\cos\gamma < 0$ is allowed. If the indication for negative $\cos\gamma$ from $B^0 \to K^{*+}\pi^-$ mentioned in Sec. IV is borne out, one should be at the verge of observing a signal, not just a lower bound, for Δm_s.

C. $K^+ \to \pi^+\nu\bar{\nu}$

A recent update of constraints on $K^+ \to \pi^+\nu\bar{\nu}$ has been published [51]. With $\cos\gamma$ constrained to lie very close to 0 on the basis of our $B \to VP$ analysis combined with the strong limit on Δm_s, the branching ratio $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ is constrained to lie not far from 10^{-10}. This is consistent with the present status of Brookhaven Experiment E787 [52], but more data are expected.

VII. CONCLUSIONS

We have studied the decays $B \to VP$, where V is a vector meson and P is a pseudoscalar meson, in a flavor-SU(3)-invariant analysis. Two main conclusions have emerged.

First, the pattern of interferences between dominant and subdominant amplitudes, particularly in the decay $B^0 \to K^{*+}\pi^-$ (discussed in Sec. IV B), favors a weak phase γ in the second quadrant of the unitarity triangle plot: $\cos\gamma < 0$. This conclusion, reached under the assumption that the tree-penguin strong phase-difference in the above process is smaller that 90°, agrees with that reached on the basis of more model-dependent analyses [5, 15, 21, 17, 18]. It depends to some extent on an estimate of the magnitude of electroweak penguin contributions, and on the assumption of flavor SU(3) in relating the penguin contributions in $B \to \phi K$ decays to those in $B \to K^*\pi$ decays. When combined with the constraint associated with Δm_s, this result favors values of γ close to 90°.

Ratios of other decay rates, including not only those listed in Table II but also the ratios $\Gamma(B^0 \to \rho^- K^+)/\Gamma(B^+ \to \rho^+ K^0)$ and $\Gamma(B^0 \to K^{*+}\pi^-)/\Gamma(B^+ \to K^{*0}\pi^+)$ (the VP analogues of the Fleischer-Mannel [8] ratio R) can shed light on tree-penguin interferences, permitting constraints on CKM phases with sufficently accurate data.

Second, there seems to be evidence for a penguin amplitude, called p'_V in our notation, at a much higher level than predicted by specific models. This amplitude contributes to a number of processes, notably $B \to K^*\eta$ and $B^+ \to \rho^+ K^0$.

We have predicted a number of $B \to VP$ decay rates to have branching ratios of a few parts in 10^6, as shown in Tables VI and VII. Assumptions made about the relative importance of gluonic and electroweak penguin contributions can be tested by measuring the ratios of certain decay rates in ρK and $K^*\pi$ channels. In particular, if the electroweak penguin amplitude responsible for the suppression of $B^+ \to \phi K^+$ is smaller than the value we have used [38], as suggested, for example, in [16], the argument for $\cos\gamma < 0$ becomes somewhat stronger.

The discovery of $B \to VP$ processes at the predicted levels would be strong evidence that the hierarchy of amplitudes suggested here should be taken seriously. One would then have a model-independent way to anticipate the strength of a whole host of B decays whose study can shed light on the source of CP violation. The advent of an upgraded detector and improved collider at the Cornell Electron Storage Rings, the debut of B-factories at SLAC and KEK, and the ability of hadron machines to contribute incisively to B physics, all make the future study of $B \to VP$ decays a potentially rich area for research.

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APPENDIX: A DICTIONARY OF AMPLITUDES

In this Appendix we give expressions for graphical amplitudes as obtained in factorization-based calculations. We use the notation of [16]. A common factor [in the SU(3) limit], $\sqrt{2}G_F m_V(\epsilon \cdot q_P)$, is omitted and we define $Q_1 \equiv -2m_\pi^2/[(m_b + m_u)(m_u + m_d)]$, $Q_5 \equiv -2m_K^2/[(m_b + m_d)(m_s + m_d)]$. We assume isospin conservation and neglect differences between Q_1 and $Q_2 \equiv -2m_\pi^2/[(m_b + m_d)(2m_d)]$ and between Q_5 and $Q_4 \equiv -2m_K^2/[(m_b + m_u)(m_s + m_u)]$. We find

$$P'_P = V_{tb}^* V_{ts} f_{K^*} F_1^{B\to\pi}(m_{K^*}^2) a_4 \ , \tag{31}$$
$$P'_V = V_{tb}^* V_{ts} f_K A_0^{B\to\rho}(m_K^2)(a_4 + a_6 Q_5) \ , \tag{32}$$
$$T'_P = -V_{ub}^* V_{us} f_{K^*} F_1^{B\to\pi}(m_{K^*}^2) a_1 \ , \tag{33}$$
$$T'_V = -V_{ub}^* V_{us} f_K A_0^{B\to\rho}(m_K^2) a_1 \ , \tag{34}$$
$$S'_P = V_{tb}^* V_{ts} f_\omega F_1^{B\to K}(m_\omega^2)(a_3 + a_5) \ , \tag{35}$$
$$S'_V = V_{tb}^* V_{ts} f_{\eta,\eta'} A_0^{B\to K^*}(m_{\eta,\eta'}^2)(a_3 - a_5) \ , \tag{36}$$
$$P'^P_{EW} = (3/2) V_{tb}^* V_{ts} f_\omega F_1^{B\to K}(m_\omega^2)(a_9 + a_7) \ , \tag{37}$$
$$P'^V_{EW} = (3/2) V_{tb}^* V_{ts} f_\pi A_0^{B\to K^*}(m_\pi^2)(a_9 - a_7) \ , \tag{38}$$
$$T_P = -V_{ub}^* V_{ud} f_\rho F_1^{B\to\pi}(m_\rho^2) a_1 \ , \tag{39}$$
$$T_V = -V_{ub}^* V_{ud} f_\pi A_0^{B\to\rho}(m_\pi^2) a_1 \ , \tag{40}$$
$$P_P = V_{tb}^* V_{td} f_\rho F_1^{B\to\pi}(m_\rho^2) a_4 \ , \tag{41}$$
$$P_V = V_{tb}^* V_{td} f_\pi A_0^{B\to\rho}(m_\pi^2)(a_4 + a_6 Q_1) \ . \tag{42}$$

In Eq. (36) the constants $f_{\eta,\eta'}$ are defined with the same normalization as f_π in terms of the quark content of the corresponding mesons.

The smallness of the amplitude P'_V in many factorized approaches [15, 16, 21, 23, 31] follows from a strong cancellation in $a_4 + a_6 Q_5$ which depends on the value chosen for m_s. (The recent treatment of [17] avoids this cancellation by choosing a small value of m_s.) The relations $P'_V = -P'_P$ and $P_V = -P_P$ are not expected to have any special significance in the factorized approach. Whereas we neglected the amplitudes S'_P and S_P, using a suppression argument based on the OZI rule for vector mesons, this property is not exhibited in the factorization approach except for a fortuitous cancellation at a particular value of N_c. This sensitivity to N_c is a measure of non-factorizing effects, and is one of the reasons one must appeal to experiment (as advocated for some amplitudes in [16] and employed more generally in [17], for example) to determine the a_i.

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